Fragment Mass Distribution of Debris

Manfred Held
Messerschmitt-Bölkow-Blohm GmbH
8898 Schrobenhausen - West Germany

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ABSTRACT

The equation established by the author for the mass distribution of the natural fragments of an explosive-filled projectile, or for that of secondary fragments behind one or several spaced target plates, can also be applied with very good results to the mass distribution of the debris from an exploded aircraft shelter.

The two constants required for this, namely, the scale parameter B and the shape parameter λ can be determined to a usually high confidence level, with a correlation coefficient close to 1, especially when the given total mass M_O is changed to a "best mass" M_{OB} that best describes the actual fragment mass distribution.

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1. INTRODUCTION

There are virtually no formulas available, at least not in unclassified published literature, that describe the mass distributon of debris from buildings, such as aircraft shelters, when several bombs have detonated inside.

The main cause of this is certainly the fact that only very few qualifield tests have been made where the mass distributions of such debris fragments have been thoroughly analyzed. The author had the opportunity of obtaining carefully recorded test results of minutely planned model aircraft shelter blasting trials <1>, and it was his intention to find out whether the mass distribution of such debris fragments could be described by a formula he had estabished earlier in context with the natural fragmentation of detonating high explosive shells.

It is demonstrated below that Held's formula, that had been established to adequately describe the mass distribution of the so-called natural fragments from high explosive projectiles, as well as that of secondary fragments (see <2> to <7>), can also be used to give a good description of the mass distribution of the debris fragments experimentally recorded in shelter blasting trials.

2. DISTRIBUTION FUNCTION

The Weibull distribution <8> can be applied to a great variety of technical problems. The distribution density of the 3-parameter Weibull distribution is as follows:

$$f(x) = \frac{\lambda(x-\mu)^{\lambda-1}}{\delta^{\lambda}} \cdot e^{-\left(\frac{x}{\delta}\right)^{\lambda}}, \quad x > 0 \tag{1}$$

with the 3 parameters

δ = scale parameter

λ = shape parameter

μ = location parameter

The 3-parameter Weibull distribution (1) reduces to a 2-parameter Weibull distribution, when the location parameter μ is set equal to 0; this is equivalent to a transformation to the new variable $x - \mu$:

$$f(x) = \frac{\lambda x^{\lambda-1}}{\delta^{\lambda}} \cdot e^{-\left(\frac{x}{\delta}\right)^{\lambda}}, \quad x > 0 \tag{2}$$

The 2-parameter Weibull distribution follows from (2) by an integration:

$$F(x) = 1 - e^{-\left(\frac{x}{\delta}\right)^{\lambda}}, \quad x > 0 \tag{3}$$

In some papers on fragment mass distribution, the 2-parameter Weibull distribution (3) is referred to as Rosin-Rammler-Sperrling (RRS) distribution, which goes back to the description of the grain size distribution in grinding processes. Sometimes, the distribution of the "fragment masses" is also termed the RRS distribution.

In context with fragmentation and the distribution of fragment sizes, the Weibull distribution (3) has entered fragmentation ballistics since Mott (see <8>). With regard to this particular application it is therefore often refer-

red to as Mott distribution, and it is usually taken to describe the distribution of the "number of fragments".

One the basis of flash X-ray pictures that permit analyzing also smaller and finer fragments, Held <4> has made an experimental approach to represent the fragment mass distribution as a function of the number of fragments.

For comparison, the 3 formulas are given below:

$$\text{RRS} : M(m) = M_0\, e^{-\left(\frac{m}{m_R}\right)^{\lambda_R}} \tag{4}$$

$$\text{MOTT} : N(m) = N_0\, e^{-\left(\frac{m}{m_M}\right)^{\lambda_M}} \tag{5}$$

$$\text{HELD} : M(n) = M_0\left(1 - e^{-Bn^{\lambda_H}}\right) \tag{6}$$

Where the symbols have the following meaning:

M_0	total mass of all fragments
$M(m)$	cumulative fragment mass, i.e. overall mass of all fragments whose mass is greater than or equal to a given mass m
$M(n)$	cumulative fragment mass, i.e. overall mass of the fragment number n, beginning with the largest fragment
m	mass of the n-th fragment
N_0	total number of fragments
$N(m)$	cumulative fragment number, i.e. number of all fragments whose mass is greater than or equal to a given fragment mass m
n	cumulative fragment number, beginning with the heaviest fragment
m_x, λ_x, B	constants.

The RRS formula is not related to the number of fragments at all, and the Mott formula requires a given number N_0 of fragments, whereas Held's formula does not need this. Any number of fine fragments may be added, even if they contribute virtually nothing to the overall mass M_0.

3. DESCRIPTION OF MASS DESTRIBUTION WITH HELD'S FORMULA

The method of how to analyze mass distributions by means of Held's formula <4> and <5> is explained below. This formula, when applied correctly, gives an excellent description of the mass distribution of the natural fragments generated in the detonation of all high explosive projectiles examined <6>, even when filled with various types of explosives, and also of secondary fragments behind a target plate, or even behind a set of multiple spaced target plates <7>. The simple equation for this is:

$$M\,(n) = M_0\,(1-e^{-Bn^{\lambda}}) \tag{7}$$

This equation is shown schematically in Fig. 1.
The constants B and λ in equation (7) are readily found by isolating the exponential term in Eq. (7)

$$\frac{M_0 - M\,(n)}{M_0} = e^{-Bn^{\lambda}} \tag{8}$$

and then taking the natural logarithm of Eq. (8):

$$\ln \frac{M_0 - M\,(n)}{M_0} = -Bn^{\lambda} \tag{9}$$

For an easy determination of the values of B and λ it is convenient to again take the logarithm of Eq. (9) so that in a logarithmic representation the point of intersection n = 1, or log n = 0, will give the constant log B directly on the ordinate axis, and the exponent λ can be determined from the slope of the straight line (Fig. 2).

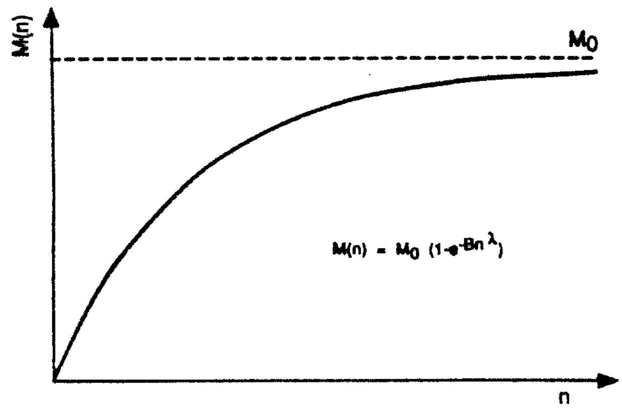

Fig. 1 Summed up mass over cumulative fragment number n.

$$\log \left(- \ln \frac{M_0 - M(n)}{M_0}\right) = \log \left(\ln \frac{M_0}{M_0 - M(n)}\right) = \log B + \lambda \cdot \log n \tag{10}$$

To this end, the value of M(n) must be computed with the associated cumulative number of fragments, n, beginning with the largest fragment. This value must then be subtracted from the total fragment mass, M_0, and then be divided by M_0. The corresponding logarithms can then be plotted in a log-log-diagram.

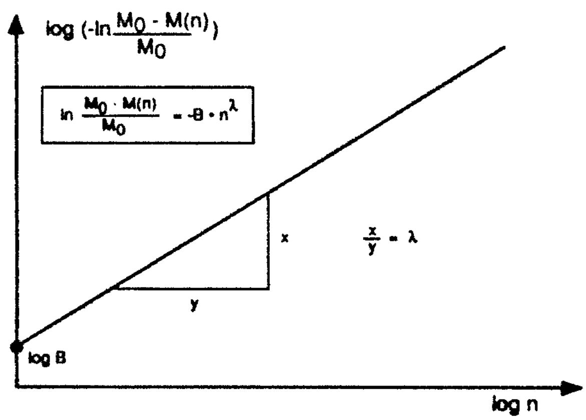

Fig. 2 Easy determination of the constants B and λ from the log-log plot.

4. FRAGMENT MASS DISTRIBUTION OF AN 155 MM HE-ROUND

Table 1 gives the natural fragments generated by the detonation of an 155 mm HE-round filled with Composition B, arranged in mass classes. For the analysis according to Held, the fragment masses M(n) must be summed up over the corresponding numbers, beginning with the largest fragment and the result must then be evaluated with Eq. (9). The total mass M_0 of the fragments is either the sum of all partial masses, which in this case is 18164 g or the total mass of the casing with 32151 g. The latter was used in the generation of the first diagram (Fig. 3).

The values obtained by the outlined method and plotted in a log-log-diagram, which is called the fragment mass distribution log-log-diagram or short FMD-log-log-diagram (Fig. 3, left) with a best-fit straight line, which gives a constant B of 0.089 and an exponent λ of 0.6531 with a correlation coefficient C of 0.9958.

Taking the derivative of Eq. (7) with respect to n gives the following equation (11) for the mass of the n-th fragment:

$$m = \frac{d\ M(n)}{dn} = M_0 B \quad \lambda n^{\lambda-1}\ e^{-Bn\lambda} \qquad (11)$$

This equation, when plotted in the diagram "mean fragment mass as a function of the cumulated number n", or short MFM-diagram, with the given M_0 and with the constants B and λ calculated, shows a not too good agreement between the numbers of fragments and the mean fragment masses in the individual mass classes according to Table 1 and Eq. (11) (see Fig. 3, right).

Table 1

Fragment Classes (g)	Number of Fragments n	Weight of Fragments in each class (g)	$\sum n$	M(n) (g)	x_1	x_2
200-250	1	205	1	205	0.99272	0.00731
150-200	1	156	2	361	0.98718	0.01290
125-150	2	260	4	621	0.97795	0.02230
100-125	6	666	10	1287	0.95430	0.04677
90-100	1	91	11	1378	0.95107	0.05016
80-90	2	175	13	1553	0.94486	0.05672
70-80	4	303	17	1856	0.93410	0.06817
60-70	15	977	32	2833	0.89941	0.10601
50-60	17	943	49	3776	0.86593	0.14395
40-50	40	1771	89	5547	0.80305	0.21934
30-40	60	1954	149	7501	0.73367	0.30970
20-30	116	2762	265	10263	0.63560	0.45319
15-20	116	2000	381	12263	0.56459	0.57166
14-15	47	683	428	12946	0.54034	0.61557
13-14	42	561	470	13507	0.52042	0.65313
12-13	59	734	529	14241	0.49435	0.70450
11-12	54	613	583	14854	0.47259	0.74953
10-11	76	801	659	15655	0.44415	0.81160
9-10	73	689	732	16344	0.41969	0.86825
8-9	100	857	832	17201	0.38926	0.94352
7-8	122	919	954	18120	0.35663	1.03107
6-7	175	1140	1129	19260	0.31615	1.15154
5-6	209	1142	1338	20402	0.27560	1.28880
4-5	310	1385	1648	21787	0.22642	1.48534
3-4	420	1455	2068	23242	0.17476	1.74433
2-3	642	1586	2710	24828	0.11845	2.13327
1.5-2.0	446	773	3156	25601	0.09100	2.39686
1.0-1.5	717	889	3873	26490	0.05944	2.82282
0.5-1.0	1102	888	4975	27378	0.02791	3.57883
0.0-0.5	4508	786	9483	28164	0	-

$$x_1 = \frac{M_0 - M(n)}{M_0} \qquad x2 = \frac{M_0}{M_0 - M(n)}$$

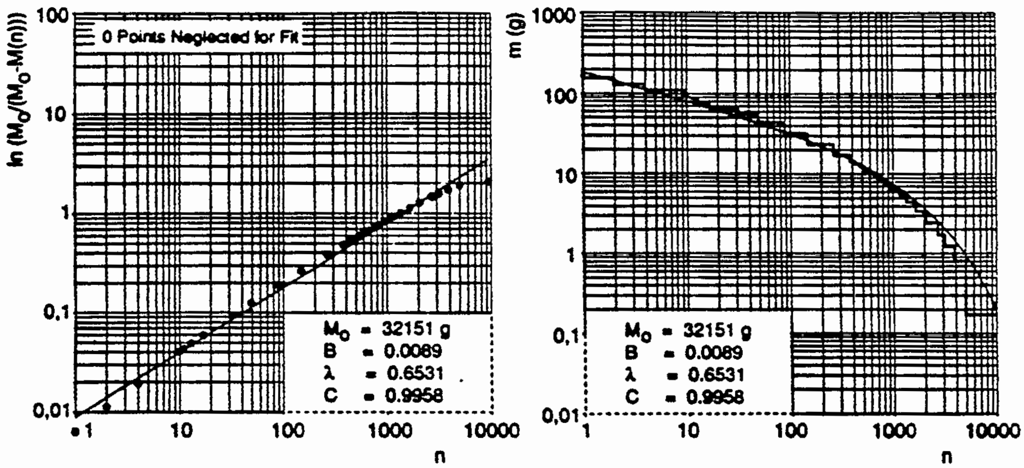

Fig. 3 Fragment mass distribution (FMD for short) log-log-diagram (left side) and mean fragment mass (MFM) (right side) as a function of the cumulative fragment number n for an 155 mm HE-projectile, filled with Composition B.

In the log-log-diagram, the straight line does not fit the measured data very well (Fig. 3 - left side). The first 4 fragments and the fragments over 4000, in particular, deviate from the straight line. Also the MFM-diagram (Fig. 3- right side) gives not a too good description of the found experimental distribution.

The agreement can be improved by adapting the overall mass M_O as well as by neglecting some of the largest fragments which do not correlate with the fragment mass distribution of the shell casing, because they originate from the end plate and from the fuze adapter flange.

Using the constants B and λ as originally determined, one can now calculate an optimum mass M_{OB}, i.e. a total mass M_{OB} which best fits this set of equations:

$$M_{OB} = \frac{M(n)}{1 - e^{-Bn^{\lambda}}} \tag{12}$$

The new constants B_B and λ_B are now determined with this new total mass M_{OB}:

$$M(n) = M_{OB} \cdot (1 - e^{-B_B \cdot n^{\lambda_B}}) \tag{13}$$

With this new total mass M_{OB} = 28318 g, which is very near on the summed up mass of the found fragment masses of 28168 g, instead of M_0 = 32151 g, which in the example given means 12% less mass, the experimental data are much better described by the fitting of a straight line. The new constants are now B_B = 0.0088 (instead of B = 0.0089) and λ_B = 0.6975 (instead of λ = 0.6531), with a correlation coefficient of 0.9994 (instead of previously 0.9958) (Fig. 4, left). As can be seen in Fig. 4, right, the cumulative number n of fragments can now be described much better as a function of the mass classes, when these constants are used.

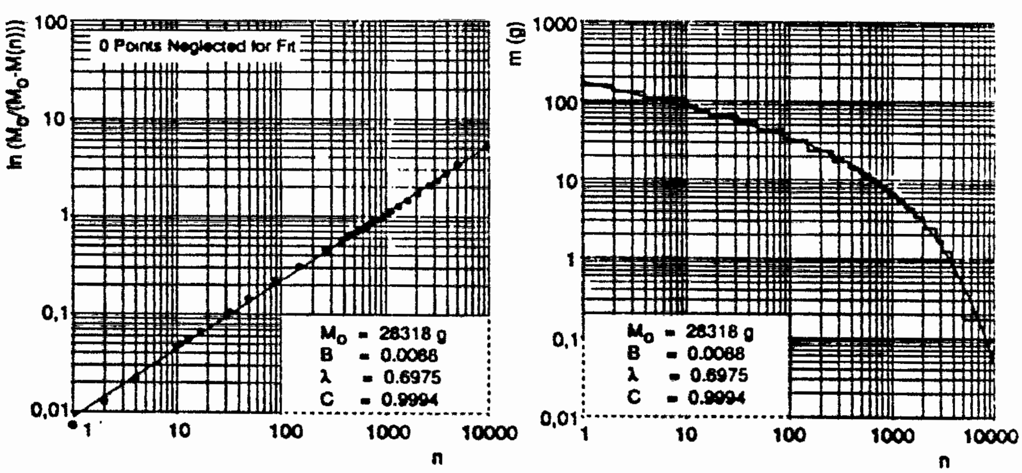

Fig. 4 FMD-log-log- and MFM-diagramm for an 155 mm HE-projectile with corrected mass compared to Fig. 3.

However, Fig. 4, left, shows that the individual points still fluctuate about the best-fit straight line, with the first 3 points - meaning the four largest fragments - deviating even more than the rest. These 4 fragments, with their random masses, must not be relevant to the fragment mass distribution.

When the first three points, corresponding to these four fragments, are omitted in this example, optimizing the mass M_0 now leads to a value of M_{OB} = 28374 g, and the constants become B_B = 0.0100 and λ = 0.6763, with a correlation coefficient of 0.9998. As can be seen in Fig. 5, left, all points - except for the three that have been purposely omitted - fit the calculated straight line rather well. Of course, the fragment mass equation with m as a function of the cumulative number of particles, n, averages the experimental values particularly well (Fig. 5, right).

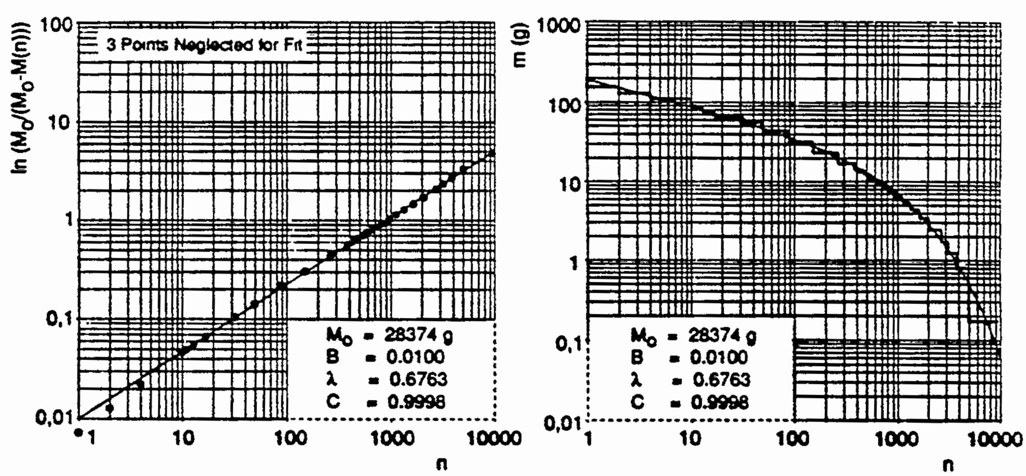

Fig. 5 FMD-log-log- and MFM-diagrams for an 155 mm HE-projectile with corrected mass, and neglecting the first 3 points (equivalent to the first 4 fragments).

5. MASS DISTRIBUTION OF DEBRIS FROM A SHELTER

The mass distribution of debris from 5 model-scale shelter trials is given in <1>, where tables 2 are presented showing the weight intervals and the associated numbers of fragments, the total weight without sieve data, and that with sieve data. As an example, Table 2 here shows Table 4-28 for the model 1 <1>.

All data presented in that paper have been analyzed using Held's formula. In this, the mass had to be optimized in order that an adequate description of the debris distribution be obtained.

Figure 6 left, shows the logarithm of the mass ratio plotted against the cumulative number of particles, n, with the given initial mass M_0 equal to 37029 kg. It is obvious from this graphic representation that the initial mass was not correct, which results in a curved line representing the fragment distribution. A straight line reduced from this diagram cannot describe the fragment masses as a function of the cumulative number (Fig. 6 right).

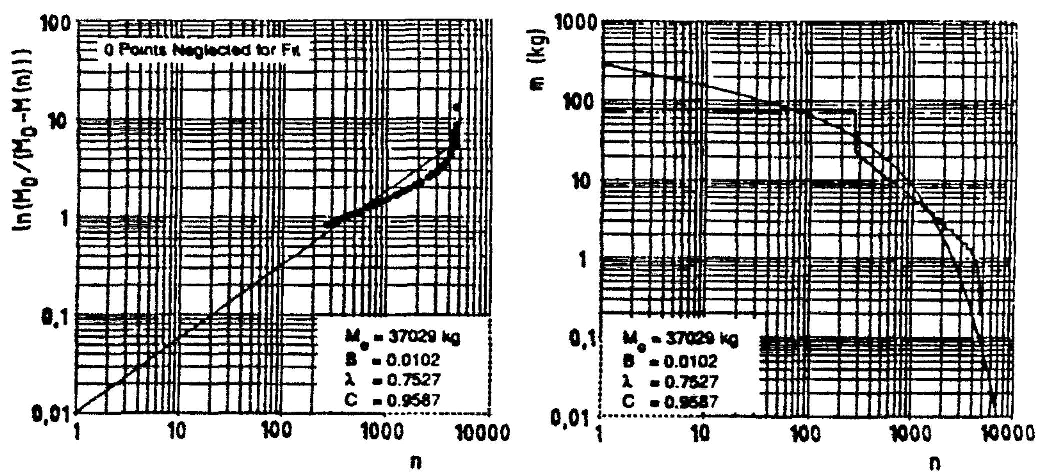

Fig. 6 FMD-log-log- and MFM-diagrams for model No. 1 without any correction.

Table 2

WEIGHT INTERVAL (LBS) W1	-	W2	WITHOUT SIEVE DATA NUMBER	TOTAL WEIGHT	WITH SIEVE DATA NUMBER	TOTAL WEIGHT
.25	-	.35	0	.00	234	73.37
.35	-	.45	31	13.67	3374	1365.94
.45	-	.55	2	.93	1294	615.95
.55	-	.65	1	.55	59	33.95
.65	-	.75	33	21.87	66	44.14
.75	-	.85	4	3.22	50	40.15
.85	-	.95	40	35.27	677	601.20
.95	-	1.05	0	.00	717	727.08
1.05	-	1.15	38	41.84	1401	1412.66
1.15	-	1.25	1	1.23	1044	1239.70
1.25	-	1.35	47	62.13	692	896.25
1.35	-	1.45	2	2.78	529	736.70
1.45	-	1.55	62	95.68	298	445.44
1.55	-	1.65	1	1.61	25	39.31
1.65	-	1.75	3	5.16	40	67.99
1.75	-	1.85	96	169.36	101	178.40
1.85	-	2.25	267	565.55	307	643.49
2.25	-	2.75	315	798.05	326	825.06
2.75	-	3.25	340	1011.48	342	1017.65
3.25	-	3.75	404	1412.13	408	1425.58
3.75	-	4.25	259	1051.65	260	1055.62
4.25	-	4.75	250	1124.14	250	1124.14
4.75	-	5.75	386	2028.23	388	2039.25
5.75	-	6.75	241	1515.13	244	1534.53
6.75	-	7.75	251	1819.89	262	1901.24
7.75	-	8.75	136	1126.12	136	1126.12
8.75	-	9.75	161	1490.10	161	1490.10
9.75	-	10.75	99	1013.62	99	1013.62
10.75	-	12.75	179	2093.33	179	2093.33
12.75	-	14.75	146	1998.09	146	1998.09
14.75	-	16.75	95	1491.29	95	1491.29
16.75	-	18.75	99	1752.39	99	1752.39
18.75	-	20.75	61	1204.03	61	1204.03
20.75	-	22.75	72	1563.89	72	1563.89
22.75	-	24.75	47	1119.02	47	1119.02
24.75	-	26.75	39	1004.29	39	1004.29
26.75	-	28.75	31	861.15	31	861.15
28.75	-	30.75	29	864.65	29	864.65
30.75	-	32.75	21	663.59	21	663.59
32.75	-	34.75	24	811.57	24	811.57
34.75	-	36.75	15	537.38	15	537.38
36.75	-	38.75	26	977.77	26	977.77
38.75	-	40.75	9	354.94	9	354.94
40.75	-	42.75	19	796.53	19	796.53
42.75	-	44.75	12	527.19	12	527.19
44.75	-	46.75	13	591.72	13	591.72
46.75	-	48.75	7	335.76	7	335.76
48.75	-	50.75	8	396.02	8	396.02
50.75	-	52.75	8	414.91	8	414.91
52.75	-	****	269	45864.77	269	45864.77
			TOTAL NB.	TOTAL WEIGHT	TOTAL NB.	TOTAL WEIGHT
			4699	81635.69	15013	89938.92

NOTE: 1 LB = 0.454 kg

If, however, the mass M_0 is optimized as outlined above, then the result is a straight line that excellently fits the measured. To this end, the total mass must be raised from 37029 kg to 40502 kg, i.e. by 8,6 % (Fig. 7, left). The resulting constants B = 0.0641 and λ = 0.4312 excellently describe the experimental fragment distribution, as can be seen in Fig. 7, right.

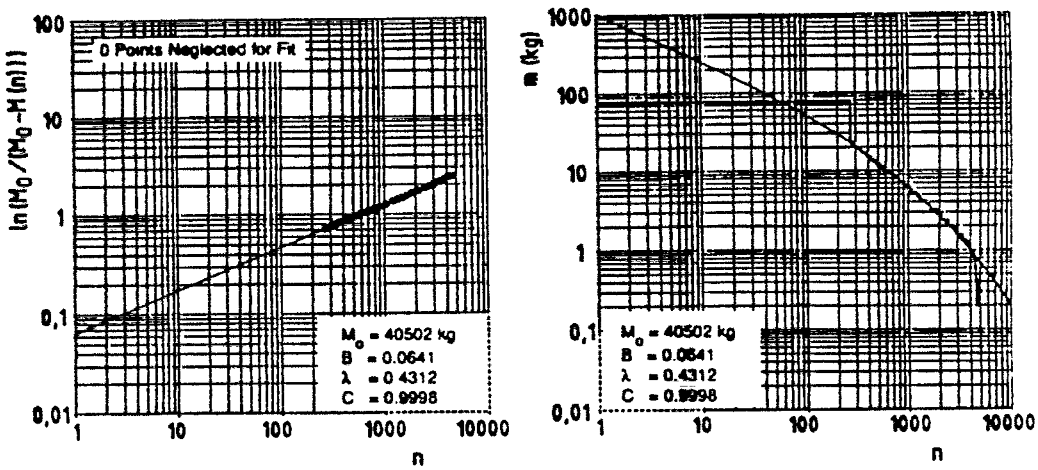

Fig. 7 FMD-log-log- and MFM-diagrams with mass correction from the data without any sieve.

With the sieve data, the mass difference is small, even though also here the initial mass M_0 of 40796 kg is not optimal (Fig. 8).

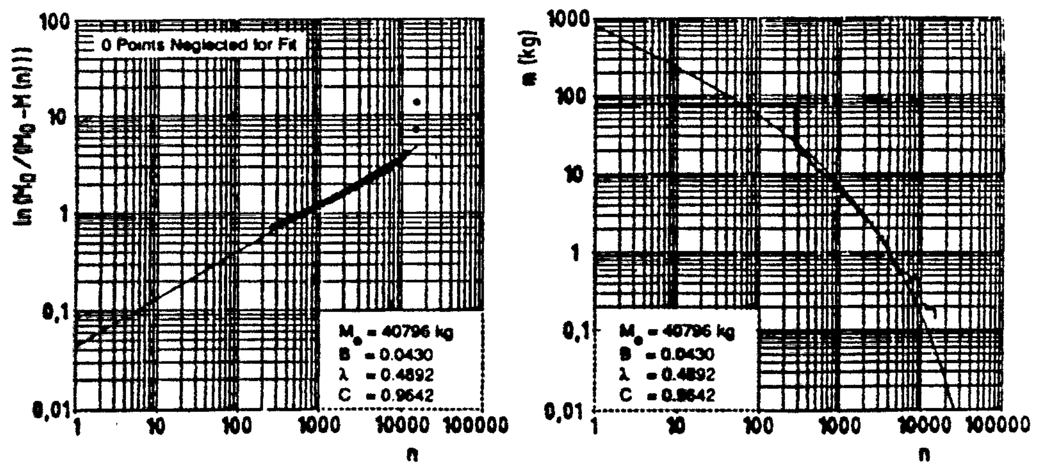

Fig. 8 FMD-log-log- and MFM-diagrams for model No. 1 with sieve data, without any correction.

An initial mass corrected by some 4 % again constitutes an optimum adaption to the mass distribution (Fig. 9).

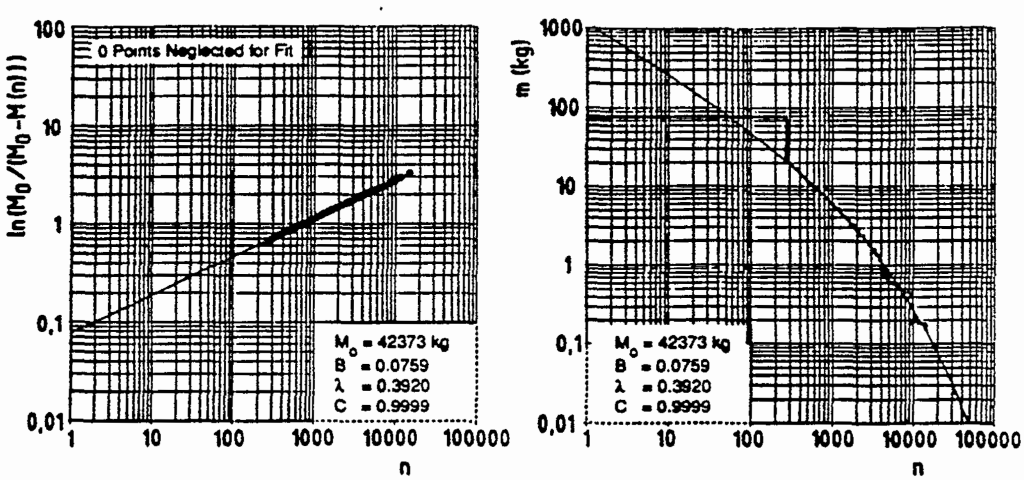

Fig. 9 FMD-log-log- and MFM-diagrams with only mass correction for the data with sieve.

With the smaller fragment masses, there is obviously an error in the analysis of the mass distribution with the sieve, which leads to a deviation of the curve fit for fragment masses of less than 0.6 grams. These small deviations exist in all analysis results with sieve data.

For reasons of space, the individual curves for the models 2 to 5 will not be presented here. With optimally selected M_0 values the curves for the fragment masses, as a function of fragment number for the 5 model tests without and with the sieving data, are compared in Fig. 10. They have indeed only relatively small deviations from one another. With the data without sieving, the mass correction is always greater than in the case with sieve data.

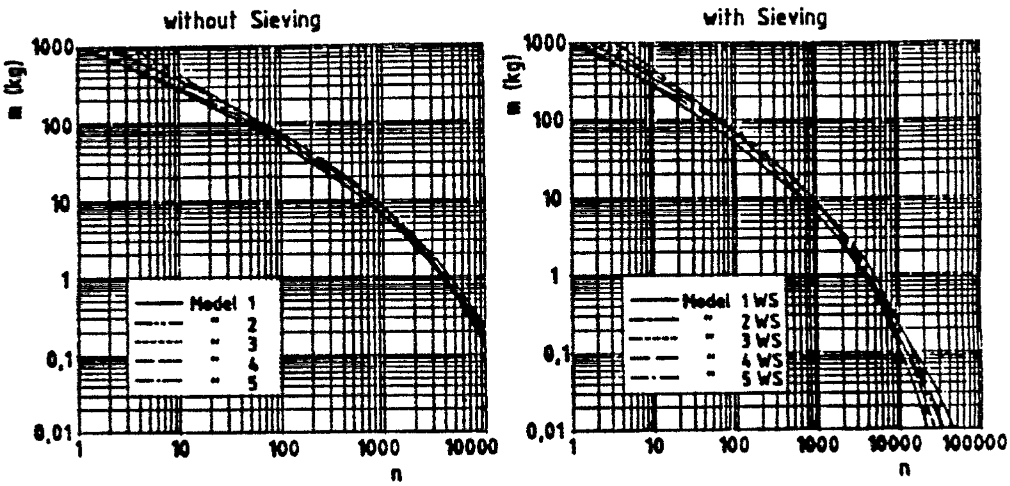

Fig. 10 Comparison of the mass distribution in the MFM-diagram for the fragment distributions of the model No. 1 to 5 without and with sieve data.

The table 3 lists the overall masses M_0 and the corrected masses $M_{0,Best}$ for the optimum fragment mass distribution, together with the constants B and λ and the correlation coefficients C, for the values without sieve data and with sieve data, for the five model tests.

Table 3

Model	M_0	$M_{0,Best}$	B	λ	C
1	37.020	40.502	0.0641	0.4312	0.9998
2	47.880	51.541	0.1363	0.3588	0.9999
3	50.582	52.874	0.0615	0.4608	0.9999
4	47.933	51.904	0.0602	0.4403	0.9999
5	41.975	44.859	0.0459	0.4932	0.9998
1	40.796	42.373	0.0759	0.3920	0.9999
2	50.644	51.461	0.1330	0.3630	0.9999
3	52.657	53.532	0.0665	0.4448	0.9999
4	50.891	51.763	0.0583	0.4458	0.9999
5	43.811	44.338	0.0404	0.5164	0.9998

Fig. 11 represents all the values of the table 3 in a graphical form. $M_{0,Best}$ varies relatively little. Also λ is fairly constant. The values of B are practically constant for tests 1, 3 and 4, but greater by a factor of 2 for test no. 2, and smaller by 30 % for test no. 5.

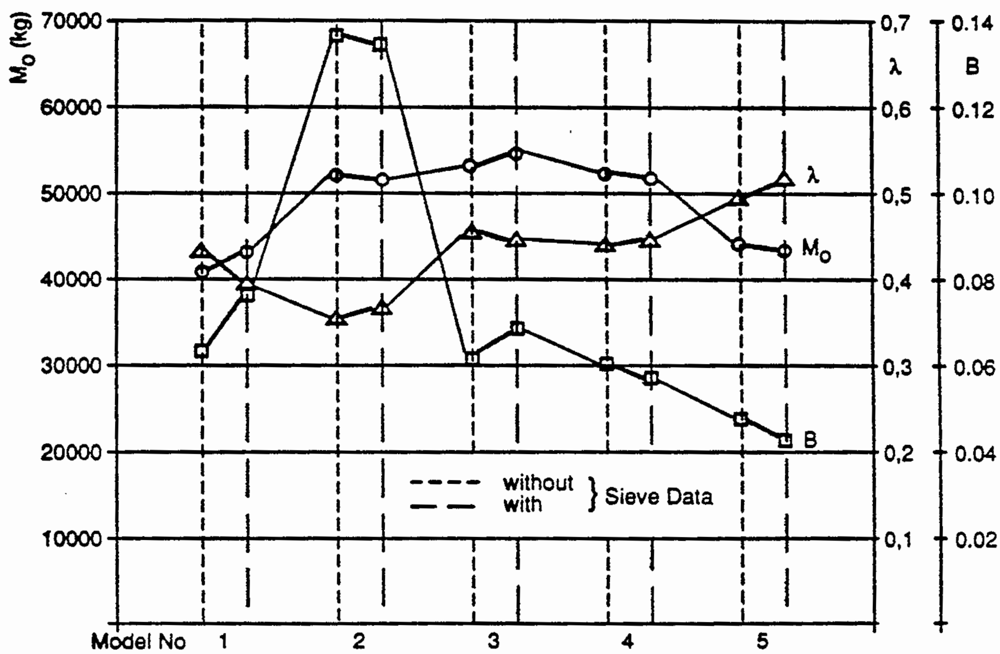

Fig. 11 The masses M_0 and the constants B and λ used for the description of the fragment mass distribution for the 5 model tests, with-out and with sieve data.

6. SUMMARY

The equation established by Held for the mass distribution of the natural fragments of an explosive-filled projectile can also be applied with very good results to the mass distribution of the debris from an exploded aircraft shelter.

The two constants required for this, namely, the scale parameter B and the shape parameter λ can be determined to a usually high confidence level, with a correlation coefficient close to 1, especially when the given total mass M_0 is changed to a "best mass" M_{0B} that best describes the actual fragment mass distribution.

The equation gives an even better description of the mass distribution of projectile - and this will show in a higher correlation coefficient - if the first, large fragments are omitted from the consideration. These fragments often do not belong into the fragment mass distribution. To omit means here that approximately 1 % to 2 % of the heaviest fragments will not be taken into consideration in the determination of the constants B and λ; this is usually done on various but reasonable grounds. Omitting certain fragments is not necessary when the mass distribution of debris fragments from an aircraft shelter is to be described.

Such an optimization in the mass distribution of

- natural fragments of an explosive-filled projectile
- debris of e.g. an aircraft shelter

according to Held produces excellent results, with correlation coefficient that usually have four nines behind the comma.

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